SOCP Radiotherapy Benchmark Test Case in CBLIB

Jagdish Ramakrishnan* Michael C. Ferris†

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Abstract

This document briefly describes how a couple of radiotherapy second-order cone program (SOCP) benchmark test cases were generated. The seven-beam and thirty-beam cases are now part of the conic benchmark library CBLIB, http://cblib.zib.de/.

The basic radiotherapy optimization problem is optimize the beam intensities delivered from various angles around the patient so as to minimize dose to healthy tissues and ensure the prescription dose to the tumor. Ideally, we would like to solve the beam angle optimization in which the optimal beam intensities together with the location of a few relevant beam angles are determined; however, the combinatorial nature of the beam selection problem makes this very challenging. One way to introduce sparse beam solutions is to sum over the beams the non-squared l^2 norm of the beam intensities from each beam, which lends itself to an SOCP formulation [1]. We use such a SOCP formulation.

We let x_{jk} be the kth beamlet intensity of the jth beam, which in full vector form we denote x. Let A be the dose deposition matrix that maps the set of all (beam, beamlet) pairs to the dose delivered to individual points on the patient (voxels). We use the following formulation

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\begin{aligned}
\underset{x\geq 0}{\text{minimize}} \quad & \sum_{j\in\mathcal{B}} \sqrt{\sum_{k\in\mathcal{O}_j} x_{jk}^2} \\
\text{subject to} \quad & d = Ax \\
& P \leq d_i \leq 1.12P, \ \forall i \in \mathcal{T} \\
& d_i \leq 1.12P, \ \forall i \notin \mathcal{T} \\
& m_l = \frac{1}{|\mathcal{V}_j|} \sum_{i\in\mathcal{V}_j} d_i, \ \forall l \\
& m_l \leq \alpha_l P, \ \forall l,
\end{aligned}
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[*] Wisconsin Institute for Discovery, University of Wisconsin–Madison, 330 North Orchard Street, Madison, Wisconsin 53715 (`jramakrishn2@wisc.edu`)

[†] Computer Sciences Department, University of Wisconsin–Madison, 1210 West Dayton Street, Madison, Wisconsin 53706 (`ferris@cs.wisc.edu`)

Structure	Number of voxels
tumor (PTV)	53,075
rectum	39,363
bladder	3,717
left femoral head	959
right femoral head	1,003
skin	15,505

Table 1: Number of voxel for structures. The bladder, left/right femoral heads, and skin were downsampled by a factor of 4, 8, and 16, respectively.

where $\mathcal{B}$ is the set of beams, $\mathcal{O}_j$ is the set of beamlets in beam j, $\mathcal{V}_j$ is the set of voxels in structure j, $\mathcal{T}$ is the set of voxels in the tumor, P is the tumor prescription dose, and α_l is a fractional constant between 0 and 1 for lth organ. The objective is essentially some measure of the total radiation delivered; intuitively, the l^1 norm over the beams introduces a sparse number of beams, and the l^2 norm over the beam intensities introduces smoothness within each beam. We have maximum dose constraints on all voxels to avoid hotspots and also upper bounds on mean dose for every critical structure.

We used the example prostate dataset provided online on http://www.cerr.info/. The dose deposition matrix A was computed by the quadratic infinite beam (QIB) pencil-beam dose calculation algorithm in CERR [2]. Table 1 provides the number of voxels for each critical structure; the bladder, left/right femoral heads, and skin were downsampled by a factor of 4, 8, and 16, respectively. The prescription dose P was set to 2 Gy, and α_l was set to 1 for the rectum and 0.8 for all other organs.

We generated a 7-beam test case and a larger 30-beam case. Each beam had approximately 70 beamlets (e.g., for the 30-beam case, there are a total of about 30×70 variables). The number of beamlets varied from beam to beam; it depended on whether the beamlet was part of the tumor when projected on to the 2D beam's eye view. Some results showing the dose wash, DVH curves, and beam sparsity compared to alternative models are provided in a poster presentation [3].

References

[1] Avishai Adler and Michael Zibulevsky. Sparse solution for the intensity-modulated radiotherapy problem using conic programming. *Technion - Computer Science Department - Technical Report*, 2009.

[2] Joseph O Deasy, Angel I Blanco, and Vanessa H Clark. Cerr: A computational environment for radiotherapy research. *Med. Phys.*, 30:979–985, 2003.

[3] Jagdish Ramakrishnan, Steven Dirkse, and Michael C. Ferris. Interfacing gams and matlab for large-scale radiotherapy optimization. http://coral.ie.lehigh.edu/ imrt/wp-content/uploads/2014/05/lehigh_poster.pdf.